Modeling and Analysis of the Interaction of Racist and Minority Populations: A Competing Species Model

A. Kazmierczak
T. H. E. Institute 1111E Brooks St. Norman, OK. 73072

Abstract

The rise of the racism in the United States has brought concern, debate, and contention to the modern world. The strategies of racists are not local and are not concentrated in major metropolitan areas. In this paper, we present a dynamical model of the interaction between racist and black populations. The formulation is based on models of interactions between competing species type dynamics. An exploration of the long-term dynamics and stability of homogeneous equilibrium solutions and their stability is given. The paper is given in multiple parts. Part two analyzes the current populations. Part three analyzes the situation when there is a rise in racists or racist sentiments are introduced into the population. Part four analyzes the scenario where more racism is added to the racist population or the racist population grows. Part five analyzes the scenario when even more racist sentiment in racist population. Part six presents conclusions.

Keywords: Racism, competing species model, equilibrium solutions, stability at equilibrium solutions.
1. Introduction

Racism is not a new phenomenon. However, there is a marked and exponential increase in the growth of racists and racism. Racism wreaks havoc to block citizens. These racists affect many areas of the economy, markets, and political and social policies. Consequently, major metropolitan areas are faced with extremely difficult, complex, and contentious political and social decisions in addressing the racism problem.

The acceptance of racism provides a Trojan horse of issues, namely, violence against blacks, inter-racial conflict. Despite these impending threats, there is not much literature that takes a dynamical systems approach to understanding the spread of racism, at a population level. Our primary objective is to bridge the gap.

In our framework, we let M represent the minority population. Of course, there will be most minorities that are peaceful and will be productive citizens. This racist population is denoted by R: R can be viewed as the total racist population of a metropolitan area. This paper is a first step in providing a mathematical modeling framework to study the evolution and interaction between these racist and black populations. The black population is modeled by standard population growth models.

The paper is organized as follows. In section 2, we develop and analyze the time-dependent autonomous black ordinary differential equation (ODE) model. We examine the equilibrium solutions, the stability of the equilibrium solutions and investigate the dynamics numerically. In section 3, we consider the situation when more racism is introduced into the system. We examine the equilibrium solutions, the stability of the equilibrium solutions and investigate the dynamics numerically for this situation. In section 4 we analyze the scenario where even more racism exists. In section 5, we
consider the scenario where even more racism exists. In section 6 we present our conclusions based on the analysis in sections 2, 3, 4, and 5.

2. Racist Minority (R, M) ODE Model

Consider the mathematical model

\[ \begin{align*}
    M &= a_1M/(1+d_1R) - a_{NR}MR/(1+d_2M) - b_1M^2 = 0 = f_N(M, R) \\
    R &= a_3R/(1+d_3M) - a_{NR}MR/(1+d_2M) - b_2R^2 = 0 = f_R(M, R)
\end{align*} \]

The populations \( R(t) \) and \( M(t) \) represent the populations of the racist and minority populations. The parameters are all assumed to be positive and their descriptions are given in Table 1a.

<table>
<thead>
<tr>
<th>Symbols</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a_1 )</td>
<td>Success rate of the black population</td>
</tr>
<tr>
<td>( a_2 )</td>
<td>Growth rate of the racist population</td>
</tr>
<tr>
<td>( b_1 )</td>
<td>Population loss in R due to intra-species competition and natural mortality</td>
</tr>
<tr>
<td>( b_2 )</td>
<td>Success loss in B due to intra-species competition and natural mortality</td>
</tr>
<tr>
<td>( a_{NR} )</td>
<td>Maximum per capita loss in R due to loss of racist sentiments</td>
</tr>
<tr>
<td>( d_1 )</td>
<td>Measures the effectiveness of R in disrupting the success rate of B</td>
</tr>
<tr>
<td>( d_2 )</td>
<td>Measures the resilience of B to strategies by R</td>
</tr>
<tr>
<td>( d_3 )</td>
<td>Measures the effectiveness of R in the success of B</td>
</tr>
</tbody>
</table>

In the case of \( d_i = b_i = 0 \), the mathematical model becomes similar to the competing species model. The parameters \( d_i \) influence the carrying capacity of the individual populations. For instance, if \( d_3 \gg 1 \) then the success rate of M is reduced. This is interpreted as a highly effective racist population, and can greatly hinder the success rate of M. The success rate of the black population depends on the unsuccessful efforts of the racist population. Notice, that if \( d_2 \gg 1 \) then the
success by Mis small, Also, if \( d_3 \gg 1 \), new blacks are introduced into the successful black population at a slower rate. The values chosen for the variables in this model are listed in Table 1b.

<table>
<thead>
<tr>
<th>( a_1 )</th>
<th>( a_2 )</th>
<th>( b_1 )</th>
<th>( b_2 )</th>
<th>( a_{NR} )</th>
<th>( d_1 )</th>
<th>( d_2 )</th>
<th>( d_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2</td>
<td>0.5</td>
<td>0.5</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

2.1 Racist Minority (M, R) ODE Model

Consider the mathematical model

\[
f_N(B, R) = M = \left( \frac{a_1}{1+d_1 R} - \frac{a_{NR}R}{1+d_2 M} - b_1 M \right) M = 0
\]

(3)

\[
f_R(B, R) = R = \left( \frac{a_3}{1+d_3 M} \right) - \frac{a_{NR}M}{1+d_2 M} - (b_2 R) \right) R = 0
\]

(4)

Since this system is nonlinear, the first step is linearization using the Jacobian.

The Jacobian for this system is defined as

\[
J = \begin{vmatrix}
\frac{\partial f_N}{\partial R} & \frac{\partial f_N}{\partial M} \\
\frac{\partial f_R}{\partial R} & \frac{\partial f_R}{\partial M}
\end{vmatrix}
\]

Taking the partial derivatives, simplifying and using the values in table for the parameters, the Jacobian becomes.

\[
J = \begin{vmatrix}
2/(1+M)- 2M/(1+2R)^2-R & -2/(1+2R)^2-2R/(1+2R) \\
-6R/(1+3R)^2-2M/(1+2R)^2 & 2/(1+3R)-2R/(1+2R)-M
\end{vmatrix}
\]
2.2 Equilibrium Points

Using the Maple CAS on (3) and (4) we obtained the following real valued equilibrium points:

\{M=0., R=0.\},
\{M=0., R=4.\},
\{M=4., R=0.\},
\{M=0.4891955799, R=0.6319394087\},
\{M=-0.4325627635, R=-0.6082709305\},
\{M=-0.4345884397, R=0.1197573734\},
\{M=-3.074988235, R=-2.874675564\},

2.3 Analyzing equilibrium points for stability

In this section we use the equilibrium points to generate the eigenvalues for the system and establish whether the equilibrium point is stable or unstable. Substituting equilibrium points into the Jacobian and solving for eigenvalues, we get the results in Table 2.

2.4 Summarization

Table 2 summarizes the results for the current population levels.

<table>
<thead>
<tr>
<th>Equilibrium Point</th>
<th>Eigenvalues</th>
<th>Node Type</th>
<th>Stability</th>
</tr>
</thead>
<tbody>
<tr>
<td>{M=0., R=0.}</td>
<td>2, 2</td>
<td>Repelling</td>
<td>Unstable</td>
</tr>
<tr>
<td>{M=0.4891955799, R=0.6319394087}</td>
<td>-5.65805218768034, 3.66988650721034</td>
<td>Saddle</td>
<td>Unstable</td>
</tr>
<tr>
<td>{M=4., R=0.},</td>
<td>-4.62387739643578, -12.9761226035642</td>
<td>Attracting Spiral</td>
<td>Asymptotically Stable</td>
</tr>
<tr>
<td>{M=-0.4325627635, R=-0.6082709305}</td>
<td>-12.9761226035642</td>
<td>Saddle</td>
<td>Unstable</td>
</tr>
<tr>
<td>{M=-3.074988235, R=-2.874675564}</td>
<td>-12.9761226035642</td>
<td>Saddle</td>
<td>Unstable</td>
</tr>
</tbody>
</table>
99, 
R=0.6319394087 

{-0.4325627635, 0.6082709305}, 
R=0.787432064951798, 0.6082709305}, 
R=0.6082709305}, 
R=0.6319394087 

{M=0.6319394087, 
R=0.6319394087 

10.980162767500+52.3825480139093*I, 10.980016276750-52.3825480139093*I 

Repelling Unstable 

{M=-0.4325627635, 0.6082709305}, 
R=0.787432064951798, 0.6082709305}, 
R=0.6319394087 

{M=0.6319394087, 
R=0.6319394087 

6.50289414872323, -0.301395706723234 

Saddle Unstable 

{M=-0.4325627635, 0.6082709305}, 
R=0.787432064951798, 0.6082709305}, 
R=0.6319394087 

{M=-0.4325627635, 0.6082709305}, 
R=0.787432064951798, 0.6082709305}, 
R=0.6319394087 

{M=0.6319394087, 
R=0.6319394087 

{M=0.6319394087, 
R=0.6319394087 

1.99897594600000+.746915672022082*I, 1.99897594600000-.746915672022082*I 

Repelling Unstable 

3. Growth of the Racist Population

In this section, we consider the situation where there is a 10% increase in the racist population. The mathematical model now becomes

\[ f_N(M, R) = M - \frac{a_1}{1 + d_1(1.1R)} - \frac{a_{NR}(1.1R)(1+d_2M)}{1+d_2M} - b_1M = 0 \] (5)

\[ f_R(M, R) = R - \frac{a_3}{1+d_3M} - \frac{a_{NR}M(1.1R)(1+d_2M)}{1+d_2M} - b_11.1R = 0 \] (6)

Using the Maple CAS on (5) and (6) we obtained the following real valued equilibrium points:

{M = 0., R = 0.},
{M = 0.3636363636},
{M = 4., R = 0.},
{M = 0.4891955799, R = 0.5744903716},
{M = 0.4325627635, R = 0.5529735732},
3.1 Analyzing equilibrium points for stability

In this section we use the equilibrium points to generate the eigenvalues for the system and establish whether the equilibrium point is stable or unstable.

3.2 Summarization

Table 3 summarizes the results for an increased racist population level.

<table>
<thead>
<tr>
<th>Equilibrium Point</th>
<th>Eigenvalues</th>
<th>Node Type</th>
<th>Stability</th>
</tr>
</thead>
<tbody>
<tr>
<td>( { M = 0., R = 0. } )</td>
<td>2, 2</td>
<td>Repelling</td>
<td>Unstable</td>
</tr>
<tr>
<td>( { M = 0., R = 3.636363636 } )</td>
<td>-5.1956747123492, 3.57341281755493</td>
<td>Saddle</td>
<td>Unstable</td>
</tr>
<tr>
<td>( { M = 4., R = 0. } )</td>
<td>-4.62387739643578, -12.9761226035642</td>
<td>Attracting Spiral</td>
<td>Asymptotically Stable</td>
</tr>
<tr>
<td>( { M = .4891955799, R = .5744903716 } )</td>
<td>.617795677261790, -.735975369361790</td>
<td>Saddle</td>
<td>Unstable</td>
</tr>
<tr>
<td>( { M = -.4325627635, R = - } )</td>
<td>39.0118243090000+140.794783474574<em>I, 39.0118243090000-140.794783474574</em>I</td>
<td>Repelling</td>
<td>Unstable</td>
</tr>
</tbody>
</table>
4. Increase of the Racist Population

In this section, we consider the situation where there is a 20% increase to the racist population. The mathematical model now becomes

\[ f_N(M, R) = M = \frac{a_1}{1+d_1(1.2R)} - \frac{a_{NR}(1.2R)/(1+d_2M)}{1+d_2M} - b_1M = 0 \]  
(7)

\[ f_R(M, R) = M = \frac{a_3}{1+d_3M} - \frac{a_{NR}(1.2R)/(1+d_2M)}{1+d_2M} - b_2(1.2R) \]  
(1.2R) = 0  
(8)

Using Maple on (7) and (8) we obtained the following real valued equilibrium points

\[
\begin{align*}
\{M &= -0.4345884397, \\
R &= 0.1088703395 \} & \quad \text{Saddle} & \quad \text{Unstable} \\
\{M &= -3.074988235, \\
R &= -2.613341422 \} & \quad \text{Repelling} & \quad \text{Unstable}
\end{align*}
\]

4.1 Analyzing equilibrium points for stability

In this section we use the equilibrium points to generate the eigenvalues for the system and establish whether the equilibrium point is stable or unstable.

\[
\{M = 0., R = 0.\},
\{M = 0., R = 3.3333333333, \}
\]
\{M = 4., R = 0.\},
\{M = .4891955799, R = .5266161739\},
\{M = -.4325627635, R = -.5068924421\},
\{M = -.4345884397, R = 0.9979781120e-1\},
\{M = -3.074988235, R = -2.395562970\}

### 4.2 Summarization

Table 4 summarizes the results for an increased police population level.

<table>
<thead>
<tr>
<th>Equilibrium Point</th>
<th>Eigenvalues</th>
<th>Node Type</th>
<th>Stability</th>
</tr>
</thead>
<tbody>
<tr>
<td>{M = 0., R = 0.}</td>
<td>2, 2</td>
<td>Repelling</td>
<td>Unstable</td>
</tr>
<tr>
<td>{M = 0., R = 3.333333333}</td>
<td>-4.80162644059918, 3.48482203321918</td>
<td>Saddle</td>
<td>Unstable</td>
</tr>
<tr>
<td>{M = 4., R = 0.}</td>
<td>-4.62387739643578, -12.9761226035642</td>
<td>Attracting spiral</td>
<td>Asymptotically Stable</td>
</tr>
<tr>
<td>{M = .4891955799, R = .5266161739}</td>
<td>.607068595230103, -.687967303230103</td>
<td>Saddle</td>
<td>Unstable</td>
</tr>
<tr>
<td>{M = -.4325627635, R = -.5068924421}</td>
<td>2250.91138756500+6580.22393699437<em>I, 2250.91138756500-6580.22393699437</em>I</td>
<td>Repelling</td>
<td>Unstable</td>
</tr>
<tr>
<td>{M = -.4345884397, R = .09979781120}</td>
<td>6.33570637219890, 0.494280168011025e-1</td>
<td>Repelling</td>
<td>Unstable</td>
</tr>
<tr>
<td>{M = -1.6823176245000+.82990103349818}</td>
<td>Repelling</td>
<td>Unstable</td>
<td></td>
</tr>
</tbody>
</table>
5. Further Increase of the Racist Population

In this section, we consider the situation where the racist population grows by 30%. The mathematical model now becomes

\[ f_N(M, R) = M = \frac{a_1}{1+d_1(1.3R)} - \frac{a_{NR}(1.3R)\left((1+d_2(M) - b_1(M))\right)}{(1+d_2(M))^2} \quad (M)= 0 \]

(9)

\[ f_R(M, R) = R = \frac{a_2}{1+d_3M} - \frac{a_{NR}(1.3R)(1+d_2(M)) - b_2(1.3R)}{(1.3R)} \quad (1.3R) = 0 \]

(10)

Using the Maple CAS on (9) and (10) and obtained the following real valued equilibrium points

\{M=0., R=0.\},
\{M=0., R=3.076923077\},
\{M=4., R=0.\},
\{M=0.4891955799, R=0.4861072375\},
\{M=-0.4325627635, R=-0.4679007157\},
\{M=-0.4345884397, R=0.09212105649\},
\{M=-3.074988235, R=-2.211288896\},

5.1 Analyzing equilibrium points for stability

In this section we use the equilibrium points to generate the eigenvalues for the system and establish whether the equilibrium point is stable or unstable.

5.2 Summarization
Table 5 summarizes the results for a zero undocumented population level.

<table>
<thead>
<tr>
<th>Equilibrium Point</th>
<th>Eigenvalues</th>
<th>Node Type</th>
<th>Stability</th>
</tr>
</thead>
<tbody>
<tr>
<td>{M=0., R=0.}</td>
<td>2, 2</td>
<td>Repelling</td>
<td>Unstable</td>
</tr>
<tr>
<td>{M=0., R=3.076923077}</td>
<td>-4.46079562308944, 3.40298046626944</td>
<td>Saddle</td>
<td>Unstable</td>
</tr>
<tr>
<td>{M=4., R=0.}</td>
<td>-8.8+3.81575680566778<em>I, -8.8-3.81575680566778</em>I</td>
<td>Attracting spiral</td>
<td>Asymptotically Stable</td>
</tr>
<tr>
<td>{M=0.4891955799, R=0.4861072375}</td>
<td>.592925463531771, -.641904798931771</td>
<td>Saddle</td>
<td>Unstable</td>
</tr>
<tr>
<td>{M=-0.4325627635, R=0.4679007157}</td>
<td>120.040189825000+323.886983609383<em>I, 120.040189825000-323.886983609383</em>I</td>
<td>Repelling</td>
<td>Unstable</td>
</tr>
<tr>
<td>{M=-0.4345884397, R=0.0921210564}</td>
<td>6.26209123066910, .19900922330896</td>
<td>Repelling</td>
<td>Unstable</td>
</tr>
<tr>
<td>{M=-3.074988235, R=2.211288896},</td>
<td>1.5567760130000+.873039656792675<em>I, 1.5567760130000-.873039656792675</em>I</td>
<td>Repelling</td>
<td>Unstable</td>
</tr>
</tbody>
</table>

6. Conclusions

In this paper we modeled and analyzed the interaction of racist and black populations. A comparison of the results in Tables 2, 3, 4, and 5 indicates that the
system is already mostly unstable. This would be a nice ideal, but it is well known that racism does exist.

It is in the nature of humans that they can be pushed, but only so hard and only so far. It is also in the nature of humans that when it goes too far, they will push back.

References:

1. In his 2009 visit to the US, the [UN] Special Rapporteur on Racism noted that "Socio-economic indicators show that poverty and race and ethnicity continue to overlap in the United States. This reality is a direct legacy of the past, in particular slavery, segregation, the forcible resettlement of Native Americans, which was confronted by the United States during the civil rights movement. However, whereas the country managed to establish equal treatment and non-discrimination in its laws, it has yet to redress the socioeconomic consequences of the historical legacy of racism." *CERD Task Force of the US Human Rights Network (August 2010). "From Civil Rights to Human Rights: Implementing US Obligations Under the International Convention on the Elimination of All forms of Racial Discrimination (ICERD)". Universal Periodic Review Joint Reports: United States of America. p. 44.


9. Coates, Ta-Nehisi (October 2017). "The First White President". The Atlantic. Retrieved June 29, 2018. It is often said that Trump has no real ideology, which is not true—his ideology is white supremacy, in all its truculent and sanctimonious power.


16. "Darity Jr., 2005"


29. James McPherson, Drawn with the Sword, page 15


36. "Barack Obama legacy: Did he improve US race relations?". BBC. Retrieved August 9, 2017
40. "Un lynchagemonestre" (September 24, 1906) Le Petit Journal
41. "DEPORTING THE NEGROES" (September 30, 1906) New York Times


59. Racial Integrity Act of 1924, Full Text at Wikisource.org


63. See: Race and health

64. In poor health: Supermarket redlining and urban nutrition, Elizabeth Eisenhauer, GeoJournal Volume 53, Number 2, February 2001


to relax lending restrictions but rather to get banks to apply the same criteria in the inner-city as in the suburbs."


82. White Americans play major role in electing the first black president, Los Angeles Times


84. "U.S. President: National: Exit Poll". CNN.


86. Long, Russ. "How to Think about Racial and Ethnic Inequality".


91. "Health of Black or African American non-Hispanic Population". CDC.


104.  Hugo Münsterberg's obituary.


107.  CCNY Archival Finding Aid, p. 81.


116. Phagan, 1987, p. 27, states that "everyone knew the identity of the lynchers" (putting the words in her father's mouth). Oney, 2003, p. 526, quotes Carl Abernathy as saying, "They'd go to a man's office and talk to him or ... see a man on the job and talk to him," and an unidentified lyncher as saying "The organization of the body was more open than mysterious."

117. "The Various Shady Lives of the Ku Klux Klan". Time magazine. April 9, 1965. An itinerant Methodist preacher named William Joseph Simmons started up the Klan again in Atlanta. On Thanksgiving Eve 1915, Simmons took 15 friends to the top of Stone Mountain, near Atlanta, built an altar on which he placed an American flag, a Bible and an unsheathed sword, set fire to a crude wooden cross, muttered a few incantations about a "practical
fraternity among men," and declared himself Imperial Wizard of the Invisible Empire of the Knights of the Ku Klux Klan.


120. Noam Chomsky, Necessary Illusions, Appendix V, Segment 20


133. "American laws against ‘coloreds’ influenced Nazi racial planners". Times of Israel. Retrieved August 26, 2017
134. United States Senate, Oversight Hearing on Trust Fund Litigation, Cobell v. Kempthorne. See also, Cobell v. Norton.


142. Jackson SA, Anderson RT, Johnson NJ, Sorlie PD (April 2000). "The relation of residential segregation to all-cause mortality: a study in black and


151. For example, Catherine A. Hansman, Leon Spencer, Dale Grant, Mary Jackson, "Beyond Diversity: Dismantling Barriers in Education," *Journal of Instructional Psychology*, March 1999.


What is Institutional and Structural Racism? ERASE RACISM


Science Review. 96 (1):
83. doi:10.1017/s0003055402004240. JSTOR 3117811.

181. Bill Quigley (July 26, 2010). "Fourteen Examples of Racism in Criminal Justice System".


192. Southern Poverty Law Center, Coloring Crime.

194. CNN, Holly Yan, Devon M. Sayers and Steve Almasy,. "Charlottesville white nationalist rally: What we know". CNN. Retrieved 2017-12-06.


